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## Equity Premium In Small Open Economy

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### **Abstract:**

It is now well known that the RBC models have enjoyed successful results in explaining the dynamics of the business cycle variables but fail to replicate similar interesting stylized facts while studying the behavior of asset prices. One line of progress for solving this shortcoming has been to modify utility to account for habit persistence and to incorporate capital adjustment costs. This paper study a small open economy general equilibrium model along with asset pricing formula based on the lognormality of the disturbance distribution. Our results stipulate that extending models with habit forming preferences and capital adjustment cost fails to account for a substantial equity premium in a small open economy environment.

**Keywords:** Asset pricing, Equity premium, habit formation, small open economy  
JEL classification: G15, G12, C63, F41

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## 1 Introduction

The literature on the behavior of asset prices in relation to consumption and other business cycle variables is relatively thin in the past years following the Mehra and Prescott (1985) seminal paper. During the 80's, general equilibrium models have enjoyed successful results in explaining the dynamics of macroeconomic aggregates; but fail to replicate similar interesting stylized facts while studying the behavior of asset prices. To some extent, the principal question of most business cycle studies in the last decades, is to reconcile the stylized facts to the economic theory and therefore to construct models with endogenous processes being able to generate the fluctuations observed in the data. Quarterly Postwar industrialized-country Data show for example that consumption is smooth while the covariance between the quarterly real consumption growth and real dividend growth is very weak. These changes in business cycle statistics have only a small effect on the equity premium.

The endowment model of Lucas (1978) first established the baselines of an abundant literature treating the relation between economic fluctuations and asset prices. Hansen and Singleton (1983) introduced a financial asset model based on consumption. As a result, the quantity of risk in the financial market is measured by the covariance between the excess stock return and consumption growth and the price of risk is the coefficient of relative risk aversion of the representative agent. However, the stylized facts<sup>70</sup> show that the average stock return is very high and the riskless interest rate is low inducing a high expected excess return on the stock. This high equity premium can only be explained with a very high coefficient of risk aversion since the covariance between stock returns and consumption is low (in the data). Mehra and

Prescott (1985) call this result the 'equity premium puzzle'.<sup>71</sup>

Kandel and Stambaugh (1991), in response to the equity premium puzzle, argue that risk aversion is much higher than what is traditionally thought. However, with very risk averse agents, the desire to transfer wealth from 'good' period (high consumption) to 'bad' period is strong. Since consumption grows steadily over time, the high risk aversion makes agents want to borrow to reduce the discrepancy between present and future consumption. Campbell (2001) shows that to reconcile this with the low observed real interest rate, we must postulate that agents are very patient. The rate of time preference is then low or even negative. Weil (1989) call this the 'risk-free rate puzzle'.

Several studies tried to resolve those enigmas. One way to do this is to introduce a class of utility function and payout structures that can generate large variability of the marginal utility of consumption. A model with a representative agent whose utility displays habit formation, introduced by Sundaresan (1989) and Constantinides (1990) produces this variability. Moreover, adding capital adjustment costs to this model can prevent households from smoothing their consumption through the capital accumulation (Jermann, 1998), and then resolve the puzzling equity premia.

Our work is related to the existing literature in two ways. First, we extend the work of Jermann (1998) and Boldrin et al. (2001) by introducing the small open economy dimension to the model. This extension introduce the behavior of the world riskfree interest rate as a new driving force of the model. Second, we introduce a 'risk premium'

<sup>70</sup>For a survey of the stylized facts related to the consumption-asset pricing framework see Campbell (2001).

<sup>71</sup>See also Cochrane and Hansen (1992) and Kocherlakota (1996) for more details about this puzzle. A brief summary of the other enigmas in literature concerns : 'the riskfree rate puzzle' of Weil (1989), 'the stock market volatility puzzle' of LeRoy and Porter (1981) and Shiller (1981) ; to quote only the most documented.

term to remove the built-in random walk property of the small open economy model.

Our purpose here is to introduce a foreign sector to the model studied by Jermann<sup>72</sup> (1998), and to allow in this way the representative household to have a financial access to the foreign economy. Incorporating the foreign sector will give another opportunity to households to smooth their consumptions. We develop a framework combining the loglinear reduced form along the lines of King, Plosser and Rebelo (1988) and the asset pricing formulae based on the lognormality of the distribution introduced by Hansen and Singleton (1983) and more recently by Campbell (1986 and 1996). This can essentially allow us, to study the behavior of the equity premium, in the case of small open economy model, with habit persistence in preferences and adjustment costs of capital. Thus, with this in mind, we study the business cycle and assets pricing implications of the model and whether see if the results obtained by the preceding studies,<sup>73</sup> and their finding hold once the foreign economy is introduced in the model.

Our results show that this model is not able to explain equity premium in a small open economy environment. This failure can be attributable to the fact that domestic households can play again on the smoothing of their consumption, with access to international financial markets. In this case the substantial addition brought to the standard RBC model by habit formation and capital adjustment costs will be canceled by opening the economy. Besides, The domestic agents can reduce fluctuations in consumption by borrowing (or lending) in bad (good) periods from the foreign financial market. Nevertheless our model can match the business cycle statistics and does better than the standard RBC model in explaining equity premia in several basis points.

<sup>72</sup>The one sector version of the RBC model with adjustment cost of capital and fixed labor.

<sup>73</sup>This include Boldrin et al. (2001), Jermann (1998), Benninga and Protopapadakis (1990), Danthine et al. (1992) and Rouwenhorst (1995) to name a random few.

The plan of the paper is as follows. In section 2 we present the model setting and discuss its solution, and section 3 examines the model predictions and presents results. Section 4 concludes.

## 2 The model

### 2.1 Model setting

We consider the case of a small open economy<sup>74</sup> with a continuum of identical infinitely lived, households. The representative agent in both countries (The home country and the rest of the world) maximizes the expected discounted sum of utility. There is a single consumption/investment good in the world which is produced according to a constant-returns-to-scale production technology by the domestic and the foreign firms, so that the import and local production are perfect substitutes. Each firm finances its investment through retained earnings.

#### 2.1.1 Firms

We suppose that the representative domestic firm, which is owned by the domestic households, has two type of purchasers, domestic and foreign customers to which it may export it's good. In each period the firm has to decide how much labor to hire and how much to invest. The manager's problem is to maximize the value of the firm to its owners<sup>75</sup>:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \{Y_{t+j} - W_{t+j}n_{t+j} - I_{t+j}\}$$

subject to the following constraints:

$$Y_t = Z_t K_t^\alpha n_t^{1-\alpha},$$

<sup>74</sup>The model is a modification of the one sector, fixed labor model.

<sup>75</sup>The value of the firm is equal to the present discount value of all current and future expected cash flows (as shown by Jermann, 1998) ; here we use the Modigliani-Miller theorem for the financing path.

where  $\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$  is the marginal rate of substitution of the household and  $n_t$  the quantity of labor input. The state of technology evolves according to the AR(1) process:

$$Z_t = \rho_z Z_{t-1} + \varepsilon_{zt}, \quad (1)$$

with  $Z_{-1}$  given and  $\varepsilon_{zt}$  is a normally distributed white noise with mean 0 and variance  $\sigma^2$  for all  $t \dots 0$ .

Prior research finds that endogenous consumption becomes even smoother as risk aversion is increased; in this way, it's more difficult to explain substantial risk premia (Rouwenhorst, 1995).

The intuition behind this is that agents can easily alter their production plans to smooth their consumption. So with this frictionless and instantaneous adjustment of the capital stock one cannot resolve this problem. Jermann (1998) suggests the introduction of capital adjustment costs to overcome this weakness. The specification of the function is the same as in Jermann (1998):

$$\phi\left(\frac{I_t}{K_t}\right) = \frac{a_1}{1-\xi} \left(\frac{I_t}{K_t}\right)^{\frac{\xi-1}{\xi}} + a_2$$

where  $\phi(\cdot)$  is a positive, concave function<sup>76</sup>. Thus, the resources allocated to investment are not transformed into next period capital with rate equal to one. The parameter  $\xi$  is the elasticity of investment,  $I_t$ , with respect to Tobin's  $q$  and  $a_1, a_2$  are chosen so as to yield a balanced growth path, for variables in the model, that is invariant to  $\xi$  (see Boldrin et al., 2001 for more details)<sup>77</sup>. The change in  $\xi$

<sup>76</sup>The concavity of the cost function captures the idea that changing the stock of capital rapidly cost more than changing it slowly (see Eisner and Strotz, 1963 and Lucas and Prescott, 1971).

<sup>77</sup>Here, as in Boldrin et al. (2001), we set  $a_1$  and  $a_2$  to be :

$$a_1 = (\exp(\bar{x}) - 1 + \delta)^{1/\xi},$$

$$a_2 = \frac{1}{\xi - 1} (1 - \delta - \exp(\bar{x})).$$

affect in a strong manner, the concavity of the adjustment costs function. Indeed as shown in figure<sup>78</sup> 1  $\phi(\cdot)$  is more concave when  $\xi$  is low.

The technology for accumulating capital is:

$$K_{t+1} = (1 - \delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t, \quad 0 < \delta < 1. \quad (2)$$

where  $\delta$  is the depreciation rate of capital.

There are no new shares issued by the firm and the capital stock is financed through retained earning (RE). This earning is defined as:

$$I_t = RE_t.$$

The domestic household has access to incomplete international financial market, because the only foreign asset they can hold is a riskfree bond whose rate of return is exogenously determined. In this case the initial conditions; in particular the initial foreign debt position of the home country; governs the steady state values of the model.

As a consequence, a random walk<sup>79</sup> component can prevent the dynamic equilibrium of the model from reaching a stable solution. To induce stationarity and remove this built-in random walk property of the model, we use an endogenous country specific risk premium term,  $\kappa_t$ , that reflects departures from uncovered interest parity<sup>80</sup> (UIP). Following Senhadji (1997), Mendoza and Uribe (2000) and Schmitt-Grohé and Uribe (2003), this risk premium term is given by:

$$\kappa_t = \exp\left(-\frac{\varphi \tilde{B}_t^*}{Y_t}\right), \quad (3)$$

where  $\tilde{B}_t^*$  is the average of aggregate foreign debt and  $\varphi$  measures the level of the risk premium. The risk premium term implies that the equilibrium is unique and induces

<sup>78</sup>The figure is retrieved from Budria, (2003) who use the same calibration and functional form as ours.

<sup>79</sup>At least one eigenvalue in the model is equal to unity.

<sup>80</sup>That is, the equilibrium steady state is unique and the model is stationary (see Dib 2003).

stationnarity in the model. At equilibrium the market clearing condition yields:

$$\tilde{B}_t^* = B_t^* \text{ for all } t.$$

There are three assets in this economy that are traded in incomplete financial markets. A perfectly divisible equity share of the representative domestic firm that is a claim to an infinite stream of firm's dividends ( $A_t$ ); so at time  $t$ , this asset delivers a payout (dividends) denoted by  $D_t$ . This asset can be purchased for  $P_t^s$  by domestic households<sup>81</sup> only. Two type of one period riskless bonds are also available, namely, domestic and foreign bonds. At the end of period  $t$  we have,

$$D_t = Y_t - I_t - W_t n_t,$$

which is the value of firm's dividends to shareholders.

### 2.1.2 Households

The representative household derives utility from consumption of a final good  $C_t$ . The preferences exhibit a simple form of habit formation, that is a stock of past consumption  $X_t$  that affects current utility:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{(C_{t+j} - X_{t+j})^{1-\gamma}}{1-\gamma} \right], \quad 0 < \beta < 1 \quad (4)$$

where  $\gamma$  is a positive parameters different from 1.<sup>82</sup> The habit stock  $X_t$  evolves as follows:

$$X_t = bC_{t-1} \quad (5)$$

we define the case where  $b > 0$  as the habit persistence preferences case. When  $b = 0$ , these preferences corresponds to those in a standard RBC model with fixed labor<sup>83</sup>.

In the case of habit persistence in utility function, the representative agent is concerned with maintaining the same level of

consumption period by period. As shown by Constantinides (1990) and Lettau and Uhlig (1997) the coefficient of relative risk aversion,  $\gamma$ , must not be high, because the relative risk aversion become more sensitive in this case. To chow this, one can compute the elasticity of intertemporal substitution (ES) and the relative risk aversion (RRA). Following Lettau and Uhlig (1997) and Allais et al. (2000), if we assume that the logarithm of consumption follows a random walk with drift:

$$C_{t+1} = g + C_t + \varepsilon_{t+1},$$

the inverse of ES is given by<sup>84</sup>:

$$\frac{1}{ES} = \left( \frac{\gamma}{1 - b \exp(-g)} \right) \left( \frac{1 + \beta b^2 \exp(-(\gamma + 1)g)}{1 - \beta b \exp(-\gamma g)} \right)$$

and the RRA is computed as:

$$RRA = \frac{\gamma}{1 - b \exp(-g) \frac{\exp(-\gamma g) - \beta \gamma}{\exp(-\gamma g) - b \gamma}}$$

<sup>81</sup>It is assumed here that foreigners purchase only the bonds denominated in their own output.

<sup>82</sup>In the special case where  $\gamma \rightarrow 1$ , the logarithmic function is obtained.

<sup>83</sup>The term  $bC_{t-1}$  can be seen as the household's habit stock, thus, b cannot be negative.

<sup>84</sup>See Lettau and Uhlig (1997) for more details.

It is evident to see that with no habit persistence ( $b=0$ ) the inverse of ES is simply equal to  $\gamma$ . Moreover, relative risk aversion is strongly related to the habit parameter<sup>85</sup>.

Domestic household has in its portfolio a share of the domestic firm and can also purchase one type of one period riskless bond  $B_t$  ( Domestic riskfree bonds)<sup>86</sup> denominated in consumption units for  $P_t^f$ . He may also make a period  $t$  acquisition of one bond  $B_t^*$  denominated on foreign output and which is redeemed for one unit of foreign output one period later. The price the household must pay for this bond is  $\kappa_t^{-1} P_t^{e*-1}$ .<sup>87</sup> Thus the price the households must pay is increasing in the foreign-debt to output ratio. With the rate of return on  $A_t$  is conditional on date  $t+1$  state of nature realization and the ones on  $B_t$  and  $B_t^*$  are not. The two riskless bonds pay one unit of the consumption good (for each) at time  $t+1$  and expire<sup>88</sup>.

Let  $O_t$  be the asset vector that contains the shares of domestic firm and the assets described above and possibly other assets so:

$$O_t = [B_t, A_t, B_t^*, \dots]$$

and  $V_t^o$ ,  $D_t^o$  be the vectors of asset prices and current period payouts respectively.

The budget constraint is:

$$O_{t+1} V_t^o + C_t = W_t n_t + O_t (V_t^o + D_t^o) \quad (6)$$

<sup>85</sup> Allais et al. (2000) compute the RRA and ES for Canada and argue that the presence of habit forming in preferences is likely to reach the value found on data, and that the model similar to what we present here account better for financial assets price changes.

<sup>86</sup> As in Jermann (1998), we suppose that the possibility of bankruptcy is excluded, so that the corporate and riskfree bonds are perfect substitutes.

<sup>87</sup> McCallum and Nelson (1998) suppose a random " Risk-premium" term that reflects temporary but persistent departures from uncovered interest parity, here we suppose instead an endogenous premium term to induce stationnarity in our small open economy model.

<sup>88</sup> Domestic riskless bonds are assumed to be in aggregate zero supply.

with  $W_t$  is the wage rate.

At date  $t$ , the representative firm does not issue new shares so that the household takes  $A_{t-1}, B_{t-1}, B_{t-1}^*, X_{t-1}$  as given and maximizes:

$$\text{Max}_{\{A_j, B_j, B_j^*, C_j, j..t\}} E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{(C_{t+j} - X_{t+j})^{1-\gamma}}{1-\gamma} \right]$$

subject to

$$X_t = bC_{t-1},$$

to the budget constraint (6) and to the technology that is specified as follows:

$$Y_t = Z_t K_t^\alpha n_t^{1-\alpha}, \quad 0 < \alpha < 1. \quad (7)$$

The gross domestic product  $Y_t$  can either be used for consumption or investment and to pay for foreign debt (or surplus).

### 2.1.3 Model solution

The usual way to resolve the general equilibrium models is to use the linearization method developed by King, Plosser and Rebelo (1988). But this method implies that expected returns are equal across securities, which mean that we cannot study the risk premium in this case. Danthine et al. (1992) show that the use of a solution technique with nonlinear functions can yield interesting results.

Following Jermann (1998), we use a combination of loglinear and lognormal environment. the solution in this case is to solve for the approximate dynamics of the model represented by a loglinear state space system of the form:

$$s_t = M s_{t-1} + \varepsilon_t,$$

with  $M$  is the square matrix that governs the dynamics of the system. This step involves the loglinearization of the first order conditions and to solve the dynamic system.

The second step makes use of the lognormal pricing formula following Hansen and Singleton (1983) and Campbell (1993).<sup>89</sup>

<sup>89</sup> As in Jermann (1998), we consider that the variables in the system are stationary.

This formula uses the fact that the random future payout of dividends can be valued by the present value relationship:

$$V_t[D_{t+k}] = \frac{\beta^k E_t[\Lambda_{t+k} D_{t+k}]}{\Lambda_t}$$

where  $\Lambda_t$  is the marginal valuation of the numeraire at t. The relation between the

dividend payout and the state vector and between the marginal valuation and the state vector pass through the factor  $l_d$  and  $l_\lambda$ :

$$\begin{aligned} \lambda_t &= l_\lambda s_t \\ d_t &= l_d s_t, \end{aligned}$$

with the error terms considered as following a multivariate normal iid processes.

## 2.2 Valuation of the Numeraire

In our model the valuation, (or the marginal utility  $h_t$ ) can be computed as follow:

$$h_t = \frac{\partial U_t}{\partial C_t} = (C_t - bC_{t-1})^{-\gamma} - \beta b E_t (C_{t+1} - bC_t)^{-\gamma}, \quad (9)$$

we can take a first order Taylor series approximation around the steady state of consumption (c), of the log of this expression:

$$h_t = \log[((1-b)c - vX)^{-\gamma} (1 - \beta b)] + \nabla' f(.) \begin{pmatrix} C_{t-1} - c \\ C_t - c \\ C_{t+1} - c \end{pmatrix} + o(2), \quad (10)$$

which can lead after some algebraic manipulations to

$$\begin{aligned} h_t = \log[(((1-b)c)^{-\gamma} (1 - \beta b))] - \frac{\gamma(1 + \beta b^2)}{(1-b)(1 - \beta b)} \left(\frac{C_t - c}{c}\right) + \\ \frac{\gamma b}{(1-b)(1 - \beta b)} \left(\frac{C_{t-1} - c}{c}\right) + \frac{\gamma \beta b}{(1-b)(1 - \beta b)} \left(\frac{C_{t+1} - c}{c}\right). \end{aligned} \quad (11)$$

With the use of the log approximation  $\frac{C_t - c}{c}$ ;  $\log(C_t) - \log(c)$  we get

$$h_t = \frac{\gamma \beta b}{(1-b)(1 - \beta b)} c_{t+1} - \frac{\gamma(1 + \beta b^2)}{(1-b)(1 - \beta b)} c_t + \frac{\gamma b}{(1-b)(1 - \beta b)} c_{t-1}. \quad (12)$$

where  $c_t$  presents the log of the time t consumption expenditure. Equation (12) approximate the marginal utility locally while ignoring the constant term in this formula when evaluating the relation between the consumption expenditure and the realized marginal utility  $h_t$ , and marginal valuation,  $\Lambda_t$ .

## 2.3 Computation of The Risk Premium<sup>90</sup>

### 2.3.1 The Value of Payout

Let  $H_{t,t+s}$  be the lifetime marginal utility of time t consumption expenditure and assume that its log ( $h_{t,t+s}$ ) is equal to a distributed lead of the log of the state vector:

$$h_{t,t+s} = \sum_{j=0}^s l_h(j) s_{t+j}. \quad (13)$$

<sup>90</sup>In what follow we use the presentation of Jermann (1998).

We evaluate the time  $t$  expectations over the lifetime marginal utility in the case of asset pricing framework,

$$E_t(H_{t,t+s}) = E_t \exp(h_{t,t+s})$$

here we assume that  $H_{t,t+s}$  is normally distributed, so the lognormality imply that:

$$E_t(H_{t,t+s}) = \exp(E_t h_{t,t+s} + \frac{1}{2} \text{var}_t(h_{t,t+s})).$$

In this case the value of a claim to a potentially random future payout  $D_{t+k}$  is:

$$V_t(D_{t+k}) = \frac{\beta^k E_t(\exp(E_{t+k}(h_{t+k,t+s+k}))D_{t+k})}{\exp(E_t h_{t,t+s})} \quad (14)$$

Making use of  $\Lambda_t = \exp(E_t h_{t,t+s})$  as the marginal valuation of the numeraire.

### 2.3.2 Expected Return and Conditional Variance

The one period holding return for assets with single payout can be defined as:

$$R_{t,t+1}(D_{t+k}) = \frac{V_{t+1}(D_{t+k})}{V_t(D_{t+k})}. \quad (15)$$

The next step is to evaluate the  $t$  period expected value of  $V_{t+1}(D_{t+k})$ , thus with the lognormality assumption we can write:

$$E_t[V_{t+1}(D_{t+k})] = \beta^{k-1} \exp[E_t(d_{t+k} + h_{t+k} - h_{t+1}) + \frac{1}{2} \text{var}_t(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1})) + \frac{1}{2} \text{var}_{t+1}(d_{t+k} + h_{t+k})],$$

which can be used to compute the formulation for the conditional expected return:

$$E_t[R_{t,t+1}(D_{t+k})] = R_{t,t+1}(1_{t+1}) \exp[-\text{cov}_t(h_{t+1}, E_{t+1}(h_{t+k} - h_{t+1})) - \text{cov}_t(h_{t+1}, E_{t+1}(d_{t+k}))]. \quad (16)$$

The RHS of this equation can be divided into three components<sup>91</sup>: the riskfree rate, the term uncertainty premium which represents the term premium for a  $k$ -periods discount bond, and the last element is the payout uncertainty premium. The risk free rate can be computed as:

$$R_{t,t+1}[1_{t+1}] = 1/V_t[1_{t+1}] = \beta^{-1} \exp(h_t - E_t h_{t+1} - \frac{1}{2} \text{var}_t(h_{t+1})). \quad (17)$$

In this work we need to define the unconditional expectation of this rate to compute the equity premium.

After making use of the lognormality assumption it is possible and useful to compute,

$$\text{var}_t(R_{t,t+1}[D_{t+k}]) = E_t(R_{t,t+1}[D_{t+k}])^2 (\exp(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}))), \quad (18)$$

which is the conditional variance of asset returns.

<sup>91</sup>See Jermann (1998) for more details about this specification.



### 3 Model Predictions

#### 3.1 Market Equilibrium

The good market clearing requires that all produced final goods are consumed, invested or used to pay capital-adjustment costs and the period asset returns. If we normalize the number of households and firms to one; then, the resource constraints holds with equality. Also the labor demand equals the labor supply. The financial market equilibrium occurs when agents hold all outstanding shares and corporate bonds<sup>92</sup>; all other assets are in zero supply. The sequence of markets equilibrium is defined as usual.

#### 3.2 Calibrating the Model

The value assigned to the model parameters are those estimated for the Canadian economy by Letendre (2003). Some other values are chosen from the literature, so that the model reproduces some features of small open economy. We consider the parameter within the range of values generally considered as linked to the habit formation (HF) case. Indeed the preference parameter  $\gamma$  is set equal to 2. As discussed in Jermann (1998) and Budria (2003) this parameter is close to risk aversion, in the case of HF preferences; Campbell (1993) estimate this value to be between 5 and 8 but the mean reversion in asset prices can increase this values up to three time (Black, 1990). Boldrin et al. (1995) consider, a value of 1 for HF case. The rate of depreciation is 0.025. The subjective discount factor is set to 0.96. The steady state value of  $n$  is equal to 0.33<sup>93</sup>; and the steady state risk premium parameter  $\varphi$  is set equal to 0.0054, that imply an average risk premium of 98 basis points at an annual rate (as in Clinton, 1998, that report estimates for Canada). The share of capital in production is

<sup>92</sup>The domestic corporate bonds are detained by domestic agents.

<sup>93</sup>So that as mentioned above the household work also, about one third of available time in steady state.

0.32. The parameter of habit persistence  $b$  is set equal to 0.58; Cochrane and Hansen (1992) use 0.5 and 0.6 for this parameter, while Constantinides (1990), requires a level of 0.8.

The elasticity of investment respect to Tobin  $q$  is estimated in the literature with values that range from 0.4 to 1.14, Abel (1980) estimate this parameter to be between 0.27 and 0.52. Jermann (1998) sets  $\xi$  equal to 0.23 which is the high adjustment costs case. We adopt this parametrization<sup>94</sup> and set  $\xi$  to 0.23. Finally the productivity shocks parameter is set to the value<sup>95</sup> 0.94436 estimated for Canada by Letendre (2003) with standard deviation of 0.00599.

The summary of these values is presented in table 1.

#### 3.3 Model Solution

The equity premium is low in the RBC model case as shown by Boldrin et al. (2001) and Jermann (1998). This result is due to the fact that in the RBC model the Sharpe ratio for equity (SR) and the standard deviation of the real return to equity,  $\sigma_{r^e}$ , are low. When the equity premium in the case of a production economy is computed as the product of the two variables,

$$E(r_{t+1}^e - r_t^f) = SR \cdot \sigma_{r^e}.$$

Indeed, the equity premium remains at zero and the result is invariant to the introduction of habit persistence in the utility function.<sup>96</sup> As discussed before the introduction of adjustment cost of capital in a model with habit preferences and fixed worked hours increases  $\sigma_{r^e}$  to a large value and this yields a substantial equity premium.

<sup>94</sup>See Prescott (1986) for a discussion of Solow residuals estimates.

<sup>96</sup>In our model the change in  $b$  let the equity with no significant change.

In the case of lognormal pricing model, dividends ( $D_t$ ) and the marginal valuation ( $\Lambda_t$ ) are lognormal. Given the joint distribution of the vector of state variables we compute the equity premium defined, as usual, as the difference between the unconditional mean equity return and the unconditional mean riskfree rate, by applying the lognormal

pricing formulae to the two rate (on equity and the riskfree rate). While the one period holding returns is defined as:

$$R_{t,t+1}^e = \frac{V_{t+1}(D_{t+k})}{V_t(D_{t+k})}$$

for holding an asset k period. This becomes  $R_{t,t+1}^f = \frac{1}{V_t(1_{t+k})}$  for the riskfree rate. This rate can be rewritten following Jermann (1998) as:

$$R_{t,t+1}^f = \beta^{-1} \exp[\lambda_t - E_t \lambda_{t+1} - \frac{1}{2} \text{var}_t(\lambda_{t+1})]$$

where  $\lambda_t$  is the logarithm of the valuation  $\Lambda_t$ . The unconditional expectation of this form can be computed as follows:

$$E(R_{t,t+1}^f) = \beta^{-1} \exp[\frac{1}{2} (\text{var}(E_t \lambda_{t+1} - \lambda_t) - \text{var}(\lambda_{t+1} - E_t \lambda_{t+1}))],$$

this value is trivially computed from the model solution. However, we can write the return to the firm equity as:

$$R_{t,t+1}^e = \frac{V_{t+1}^D + D_{t+1}}{V_t^D}$$

in which case the analytical closed form for the unconditional expectation of this return is quite difficult to compute. As shown by Jermann (1998) we need simulation to find this unconditional mean.

The results we obtain show that the equity premium computed using the lognormal formulae is about 0.025 % (2.5 basis points annually), which is low compared to the premium obtained with historical data, for example Allais et al. (2000) obtain an equity premium of 3.47 for Canada. The summary of the results is presented in table 3. In Table 2 the returns for the period 1984Q4-2002Q4 are presented using Canadian data for comparison purpose.

That is, even with habit persistence in preferences and adjustment costs of capital the model fails to account for a substantial equity premia when the foreign sector is introduced. This failure can be explained by the

smoothing of consumption by the households when confronted to fluctuations in their consumption plans. In this case they can borrow or lend to foreign country to have the same level of consumption. As explained before, with habit persistence, economic agents are not concerned only by the actual level of consumption, but they aim to maintain the same level of consumption period by period.

Despite this failure, the model arrives to give an equity premium that is high compared to the one obtained by standard RBC model (which gives no equity). The model is also, able to match a selected business cycle statistics. For example, the standard deviation of output is at 1.78 when it is about 1.72 in the Canadian data. the relative deviation between consumption and output is at 0.179 while the data give 0.54, this can be explained by the smoothness of the

consumption in the model. The consumption is three time less volatile and it's about 0.32 compared to the value of 0.93 obtained in the data.

This results show that what Jermann (1998) certifies as a performance for a RBC model augmented with habit formation and adjustment costs, can easily fail to account for asset pricing statistics, when introducing a new element on this model (access to foreign credits). As discussed in Abel (1991), the volatility of the interest rate is too high, this is one problem with habit persistence that makes this rate too volatile. The other problem is that habit formation preferences display a strong aversion to intertemporal substitution this leads to high variation in interest rate (Jermann, 1998).

The second part of our analysis concerns the impulse response functions. Figure 2 and 3 display impulse response of the variables in the models to a unit positive productivity shocks. The output and investment responses to a 1 percent positive productivity impulse are standard in this kind of models. The dividend response shows that the dividends are procyclical, this is the same thing as in the closed economy version of the model. Indeed Jermann (1998) found that, even, with and without habit, dividends are more procyclical with adjustment cost of capital. The marginal utility response is also in lines of closed model, and the response is negatively serially correlated with a hump-shaped response to a technology shocks. The consumption displays also a hump-shaped response because, under habit formation, households smooth both the level and the change of consumption. The pick of the consumption and marginal utility responses take place after about 10 quarters. The responses for other variables to a 1% technology shocks are common standard as in the literature.

#### 4 Concluding Remarks

Prior research on endowment model following Lucas (1978) and Campbell (1986) has focussed on various modifications of a standard RBC model in an effort to resolve its puzzling pricing implications. Models with trivial production sectors, habit persistence in preferences imposed to smooth consumption, and adjustment costs of capital assumed to increase the volatility of returns, have the potential to account for the equity premium and other asset pricing components.

Nevertheless the same model, augmented with a foreign sector, fails to generate a substantial equity premium and to explain the equity generated by the historical data in case of small open economy. Indeed, in this paper, using the lognormal-loglinear model solution, we evaluate asset prices in small open economy and highlight some shortcomings. First, as discussed above, the model generates a low risk premia. The second shortcoming of the model is that, consistent with the finding of Heaton (1995) and Boldrin et al. (2001) the volatility of the riskfree (and risky) rate is too high, This is a typical problem for the utility functions displaying habit formation. Habit persistence makes marginal utility very volatile even for smooth consumption profiles (Budria, 2003). This creates a large swinging movement in the expected marginal utility at successive dates. This implies large movements in the riskfree rate.

In sum, this model does well when the results are compared to the business cycle statistics. Some features imposing more constraints on borrowing from the rest of the world, and then preventing the household from smoothing consumption, are needed to improve the performances of this model and replicate equity premium obtained with the closed economy version of the model. We will address this interesting question in future research.

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<b>Table 1: Model Calibration</b>										
<b>Parameter</b>	$\rho_z$	$\sigma_z$	$\beta$	$\bar{x}$	$\alpha$	b	$\delta$	$\gamma$	$\zeta$	$\varphi$
<b>value assigned</b>	0.94436	0.00599	0.96	0.0040	0.32	0.58	0.025	2.0	0.23	0.006

<b>Table 2: Returns for the period 1984:4-2002:4</b>		
<b>Asset</b>	<b>Mean*</b>	<b>Variance</b>
Domestic short-term bond return (CA T-bills)	8.1049	11.93544
Foreign short-term bond return (US T-bills)	5.2324	3.01882
Stock return (S&P/TSX60)	3.4497	0.00076

☐ The quarterly returns (percent) are converted from monthly data for USA and Canada for the period 1984:4-2002:4. The asset return Data is annualized.

<b>Table 3: Equity Premium Statistics</b>							
	$E(r^e - r^f)$	$\sigma(r^e - r^f)$	$\sigma(r^e)$	$\sigma(r^f)$	$\sigma(\Delta \ln(C))$	$\rho(r^e - r^f, \Delta \ln(C))$	$\text{cov}(r^e - r^f, \Delta \ln(C))$
<b>Model</b>	0.0247	79.252	78.864	7.058	0.6822	0.004782	0.2584
<b>Data*</b>	3.47	15.53	na	na	2.04	0.33	10.58

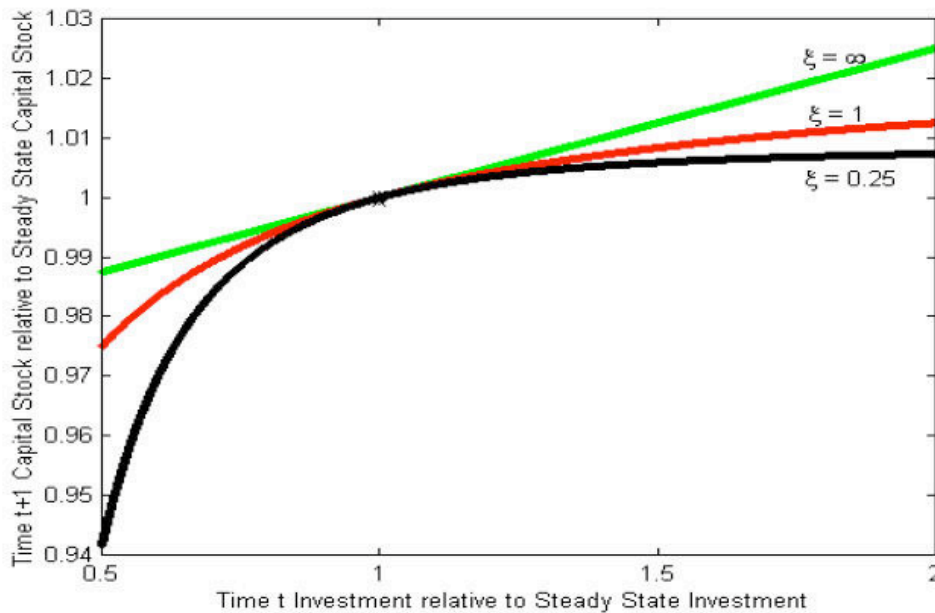
☐ Data statistics are from Allais et al.(2000) (we only report the canadian stat.). the first column represents the average excess return. The second and fifth column are the standard-errors of the excess return and the

consumption growth. In the last two column the covariance and the correlation coefficient between the excess return and the consumption growth is represented. Moments are averages of 100 replications of length 500.

Table 4: Business cycle Statistics									
Statistics	$\sigma_Y$	$\sigma_C$	$\sigma_I$	$\sigma_P^e$	$\sigma_D$	$\sigma_C/\sigma_Y$	$\sigma_I/\sigma_Y$	$\rho(Y,C)$	$\rho(Y,I)$
Model	1.78003	0.31971	2.18099	1.50586	1.52692	0.17961	1.22526	0.88255	0.99038
Data*	1.72	0.93	5.13	na	na	0.54	2.98	0.80	0.77

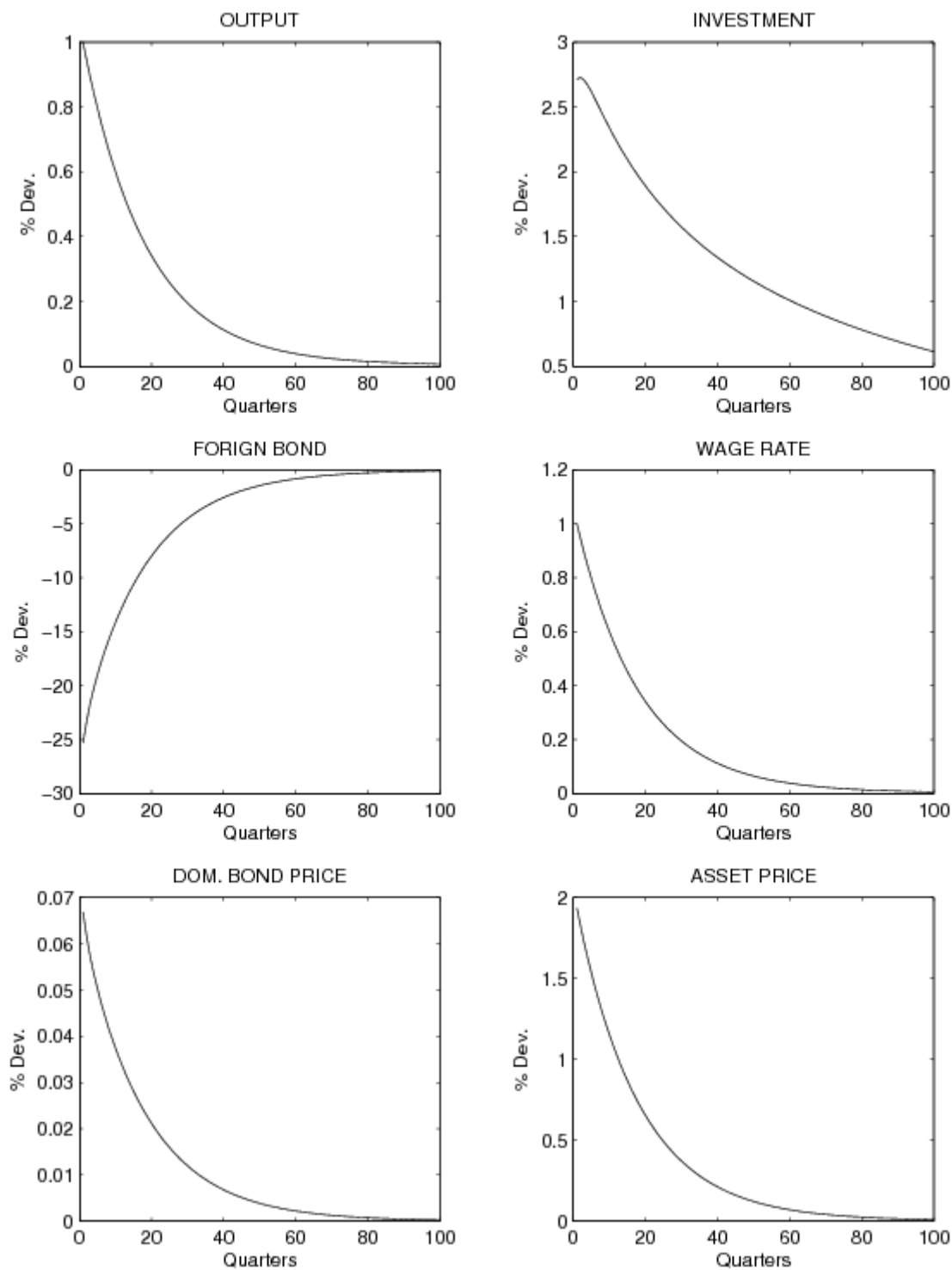
\* The Data statistics are taken from Letendre (2003). This study use quarterly Canadian Data (from 1981Q1 to 2001Q4) filtered with HP filter (we use the same filter to compute the moments of the model).

Figure 1: The Cost of Adjustment Function\*



\* Source: Budria (2003).

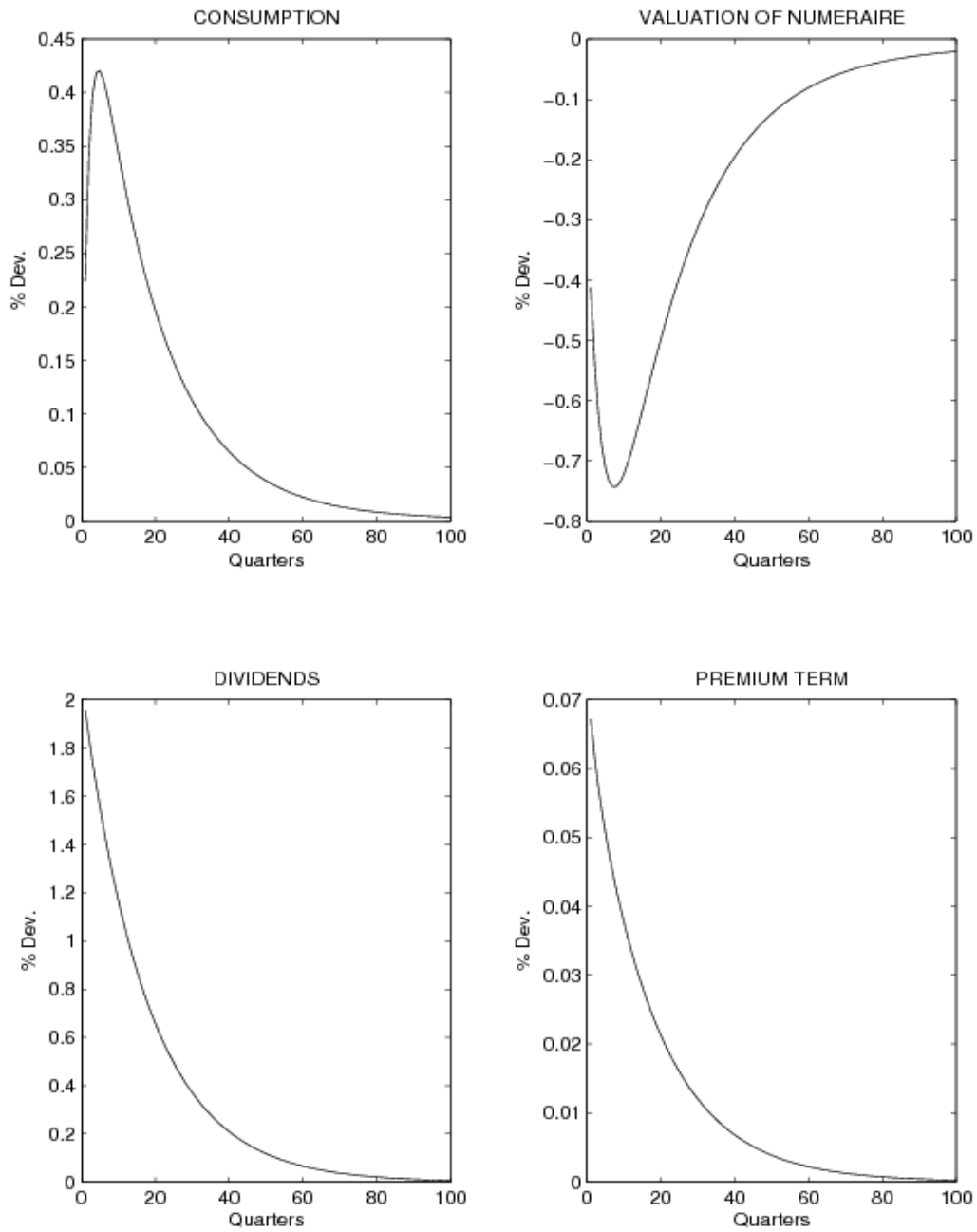
**Figure 2:** Impulse Response<sup>‡</sup> Functions to a Unit Technology Shock



<sup>‡</sup> The impulse is a 1% positive productivity shock, the responses are in % deviations from steady state values.



**Figure 3:** Impulse Response Functions to a Unit Technology Shock



Note : The valuation of numeraire is equivalent here to the marginal utility.